Explaining SMART and GSIM

Olivier Jammes
Marcelo Olarreaga

The World Bank

WORK IN PROGRESS

April 2005

We first introduced the simplest version of SMART and its definition of trade creation and trade diversion. We then relax assumption 3) below (i.e., infinitely elastic export supply functions) and re-derive the trade creation and trade diversion terms. We finally turn into GSIM, which is the more recent multi-market extension of SMART.

1. Simplest Version

Assumptions:

1) *Partial Equilibrium*: no income effects

2) *Armington Assumption*: HS 6 digit goods imported from different countries are imperfect substitutes, i.e., bananas from Ecuador are an imperfect substitute to bananas from Saint Lucia.

3) *Export supplies are perfectly elastic*: world prices of each variety (e.g., bananas from Ecuador) are given.

Analytical setup

One possible analytical setup for the demand structure in SMART is to assume a two-stage budgeting procedure (where income is kept exogenous). A better alternative is to assume a quasi-linear an additive utility function that is also additive on a composite numéraire good. More formally:

$$U = \sum_g u_g\left(m_g\right) + n \tag{1}$$

where n is the consumption of the composite numéraire good, m_g is the consumption of the aggregate import good g (aggregate in the sense that it is a function of imports of good g from different countries); and u_g is the sub-utility function of good g. The fact that the utility function is additive ensures that there are not substitution effects across goods g, and the linearity on the composite and numéraire good n ensures that there are no income effects.

Maximization of (1) subject to a budget constraint yields:

$$
\begin{aligned}
m_{g,c} &= f\left(p_{g,c}^{d}; p_{g,\neq c}^{d}\right), \forall g, c \\
n &= y - \sum_{c}\sum_{g} p_{g,c}^{d} m_{g,c}
\end{aligned}
\tag{2}
$$

where $m_{g,c}$ are imports of good g from country c, $p_{g,c}^{d}$ is the domestic price of imported good g from country c, $p_{g,\neq c}^{d}$ is the domestic price of good g imported from all countries other than c, y is national income. Thus consumption of the composite and numéraire good, n absorbs all income effects.

Domestic prices are given by:

$$
p_{g,c}^{d} = p_{g,c}^{w}\left(1 + t_{g,c}\right)
\tag{3}
$$

where $p_{g,c}^{w}$ is the world price of good g imported from c, $t_{g,c}$ is the tariff imposed on imports of good g imported from c, and is defined as:

$$
t_{g,c} = t_{g}^{MFN}\left(1 - \theta_{g,c}\right)
\tag{4}
$$

where t_{g}^{MFN} is the Most Favored Nation (MFN) tariff imposed on good g, and $\theta_{g,c}$ is the tariff preference ratio on good g when imported from country c.[1]

---

[1] By (4), $\theta_{g,c} = 1 - t_{g,c}/t_{g}^{MFN}$.

Trade creation

Trade creation is defined as the direct increase in imports following a reduction on the tariff imposed on good g from country c. To obtain this, SMART uses the definition of price elasticity of import demand:

$$\varepsilon_{g,c} = \frac{dm_{g,c}/m_{g,c}}{dp^d_{g,c}/p^d_{g,c}} < 0 \tag{5}$$

Solving (5) for $dm_{g,c}$ we obtain the trade creation ($TC_{g,c}$) evaluated at world prices and associated with the tariff reduction on good g when imported from country c:[2]

$$TC_{g,c} = p^w_{g,c} dm_{g,c} = p^w_{g,c} \varepsilon_{g,c} m_{g,c} \frac{dp^d_{g,c}}{p^d_{g,c}} \tag{6}$$

Note that using (3), we have $dp^d_{g,c} = p^w_{g,c} dt_{g,c}$. Substituting this and (3) into (6) yields:

$$TC_{g,c} = p^w_{g,c} dm_{g,c} = p^w_{g,c} \varepsilon_{g,c} m_{g,c} \frac{dt_{g,c}}{(1+t_{g,c})} = \varepsilon_{g,c} m_{g,c} \frac{dt_{g,c}}{(1+t_{g,c})} \tag{7}$$

Equation (7) defines the extent of trade creation on imports of good g from country c.

Note that in the last equality we simply choose units of all goods so that the world prices are equal to 1. One can then interpret $m_{g,c}$ as import value of good g from country c measured at world prices. This normalization of units is undertaken from now on in order to simplify the expressions, so that $m_{g,c}$ represents both imported quantities and value of good g from country c. As long as world prices are kept exogenous (i.e., export supply functions are perfectly elastic), this normalization has no implications for the derivations above and below.

---

[2] Recall that world prices are assumed to be fixed given the assumption of perfectly elastic export supplies in every country c for every good g.

To obtain the overall level of trade creation across goods or countries one simply needs to sum equation (7) along the relevant dimensions:

$$TC_g = \sum_c TC_{g,c}$$

$$TC_c = \sum_g TC_{g,c} \qquad (8)$$

$$TC = \sum_g \sum_c TC_{g,c}$$

Trade diversion

If the tariff reduction on good g from country c is a preferential tariff reduction (i.e., it does not apply to other countries, $\neq c$, then imports from country), then imports of good g from country c are further going to increase due to the substitution away from imports of good g from other countries that becomes relatively more expensive. This is the definition of trade diversion in the SMART model.

In order to measure trade diversion, let us use the definition of the elasticity of substitution, $(\sigma_{g,c,\neq c})$ across imports of good g from country c and all other countries ($\neq c$):

$$\sigma_{g,c,\neq c} = \frac{d\left(\dfrac{m_{g,c}}{m_{g,\neq c}}\right) \Big/ \dfrac{m_{g,c}}{m_{g,\neq c}}}{d\left(\dfrac{p_{g,c}^d}{p_{g,\neq c}^d}\right) \Big/ \dfrac{p_{g,c}^d}{p_{g,\neq c}^d}} < 0 \qquad (9)$$

Note that:

$$d\left(\frac{p_{g,c}^d}{p_{g,\neq c}^d}\right) \Big/ \frac{p_{g,c}^d}{p_{g,\neq c}^d} = \frac{\dfrac{p_{g,c}^w dt_{g,c}}{p_{g,\neq c}^w\left(1+t_{g,\neq c}\right)}}{\dfrac{p_{g,c}^w\left(1+t_{g,c}\right)}{p_{g,\neq c}^w\left(1+t_{g,\neq c}\right)}} = \frac{p_{g,c}^w dt_{g,c}}{p_{g,c}^w\left(1+t_{g,c}\right)} = \frac{dt_{g,c}}{\left(1+t_{g,c}\right)} \qquad (10)$$

Recalling that by definition of trade diversion $dm_{g,c} = -dm_{g,\neq c}$, we have:

$$d\left(\frac{m_{g,c}}{m_{g,\neq c}}\right) = \frac{dm_{g,c}}{m_{g,\neq c}} - \frac{m_{g,c}\,dm_{g,\neq c}}{m_{g,\neq c}^2} = \frac{dm_{g,c}\left(m_{g,c} + m_{g,\neq c}\right)}{m_{g,\neq c}^2} \tag{11}$$

Substituting (11) and (10) into (9) and solving for $dm_{g,c}$ yields the expression for trade diversion, $TD_{g,c}$:

$$TD_{g,c} = dm_{g,c} = \frac{m_{g,\neq c}\,m_{g,c}}{m_{g,\neq c} + m_{g,c}} \frac{dt_{g,c}}{1+t_{g,c}} \sigma_{g,c,\neq c} \tag{12}$$

Constraining Trade Diversion

There is one additional problem associated with the measurement of trade diversion. Indeed, by definition of trade diversion it cannot be larger than the original imports of good g from other countries $\neq c$, i.e., $TD_{g,c} = dm_{g,c} = -dm_{g,\neq c} \leq m_{g,\neq c}$. A simple way of introducing this constraint is to defined trade diversion as as follows:

$$TD_{g,c} = dm_{g,c} = -dm_{g,\neq c} = \begin{cases} \dfrac{m_{g,\neq c}\,m_{g,c}}{m_{g,\neq c} + m_{g,c}} \dfrac{dt_{g,c}}{1+t_{g,c}} \sigma_{g,c,\neq c} & \text{if } -dm_{g,\neq c} \leq m_{g,\neq c} \\ m_{g,\neq c} & \text{if } -dm_{g,\neq c} > m_{g,\neq c} \end{cases} \tag{13}$$

So the constraint is binding only when it is necessary.

An alternative to the simple constraint in (13) is the one currently used by SMART. It introduces the constraint for all observations independently of whether the constraint is binding or not. This is done by transforming (12), so that $TD_{g,c} = dm_{g,c} \leq m_{g,\neq c}, \forall g, c$:

$$TD_{g,c} = dm_{g,c} = -dm_{g,\neq c} = \frac{m_{g,\neq c}\,m_{g,c}\dfrac{dt_{g,c}}{1+t_{g,c}}\sigma_{g,c,\neq c}}{m_{g,\neq c} + m_{g,c} + \left[m_{g,c}\dfrac{dt_{g,c}}{1+t_{g,c}}\sigma_{g,c,\neq c}\right]} \tag{14}$$

By adding the term in (14) the term in square brackets to equation (12), SMART constraints trade diversion to be equal to $m_{g,\neq c}$ when the term in square brackets (the

change in tariffs multiplied by the change in relative prices and the elasticity of substitution) tends to infinity (or minus infinity). Indeed:

$$\lim_{\left[m_{g,c} \frac{dt_{g,c}}{1+t_{g,c}} \sigma_{g,c,\neq c} \right] \to -\infty} TD_{g,c} = m_{g,\neq c} \tag{15}$$

Equation (14) is clearly an underestimation of the trade diversion effect (we add a positive term to the denominator), whenever the term in squared brackets does not tend to infinity (e.g., for small tariff changes). More problematic is the fact that the terms in square brackets cannot tend to infinity unless either imports from c $\left(m_{g,c} \right)$ or the elasticity of substitution are initially infinitely large. In which there is either no reason to worry about trade diversion or we are in a world with perfectly homogeneous goods in which case the constraint is always binding. Under more reasonable assumptions, the term in squared brackets can only tend to $-m_{g,c} \dfrac{t_{g,c}}{1+t_{g,c}} \sigma_{g,c,\neq c}$ as $dt_{g,c}$ tends to $-t_{g,c}$

when the tariff on good g from country c is eliminated. It is then not clear to which value the trade diversion term tends to, a part from the fact that it is clearly an underestimation of the true trade diversion for most values. For these reasons, we suggest the use of (13) rather than (14) to measure trade diversion.

Again the expression in (13) or (14) could be added across different dimensions (goods, countries or both) to obtain total trade diversion terms as we did for trade creation in equation (8). Finally, the total increase in exports of good g from country c associated with a preferential tariff granted to good g originating in country c is given by the sum of the trade diversion and trade creation terms.

2. More complex version

An important assumption so far is that export supply functions are infinitely elastic. Thus the price of good g from country c is exogenously given. This could be quite an awkward assumption. For example if your experiment considers Brazil granting preferential treatment to milk exporters from Uruguay, this implicitly assumes that Uruguayan exporters would continue to export milk at the same price and in sufficient quantities as

to satisfy Brazilian demands for Uruguayan milk. A casual look at milk production in Uruguay and consumption in Brazil suggests that the full Uruguayan milk production may be sufficient to satisfy only a couple of supermarkets in Sao Paolo. Thus the assumption that prices will be given seems unrealistic.[3]

The more complex version of SMART introduces upward sloping export supply functions.

New Set of Assumptions:
1) *Partial Equilibrium*: no income effects
2) *Armington Assumption*: HS 6 digit goods imported from different countries are imperfect substitutes, i.e., bananas from Ecuador are an imperfect substitute to bananas from Saint Lucia.
3) *Export supplies are upward sloping*: world prices of each variety (e.g., bananas from Ecuador) are endogenous. (one disturbing assumption here is that bananas exported from Ecuador to the USA are not substitutable with bananas exported to the European Union. To change this, one either needs to model the world market for Ecuadorian bananas; that's the reason why GSIM was developed by Francois and Hall (2003); an alternative to GSIM is to follow the world market specification of Hoekman, Ng and Olarreaga (2004), or assume an Armington structure on the export side too; these alternatives are explained in the next sections).

Thus assuming that export supplies are not infinitely elastic, one can easily obtain the modify formula for trade creation by recognizing that world prices (or prices received by exporters of good g from country c) are likely to be endogenous. To do so, let us first define the export supply elasticity of good g from country c as:

$$\mu_{g,c} = \frac{dx_{g,c}/x_{g,c}}{dp^w_{g,c}/p^w_{g,c}} > 0 \tag{16}$$

---

[3] Note it may be a much more realistic assumption if one considers Burundi granting preferential access to European producers of milk, given the much larger size of Europe relative to Burundi's market.

Note that by definition $dx_{g,c} = dm_{g,c}$. Thus solving (16) for $dp_{g,c}^w$, we have:

$$dp_{g,c}^w = \frac{dx_{g,c}/x_{g,c}}{\mu_{g,c}/p_{g,c}^w} = \frac{dx_{g,c}/x_{g,c}}{\mu_{g,c}} \tag{17}$$

where the second equality is simply explained by the fact that world prices are initially normalized so that they all equal 1 (see page 3 for a discussion).

Now using the definition of domestic prices in (3), we have that:

$$dp_{g,c}^d = p_{g,c}^w dt_{g,c} + dp_{g,c}^w \left(1 + t_{g,c}\right) \tag{18}$$

Trade creation

Substituting (17) and (18) into the definition of trade creation given by (6), and recalling that initial world prices have been all normalized to unity, yields:

$$TC_{g,c} = dm_{g,c} = dx_{g,c} = \varepsilon_{g,c} m_{g,c} \frac{dt_{g,c}}{1 + t_{g,c}} \left(\frac{1}{1 - \varepsilon_{g,c}/\mu_{g,c}} \right) \tag{19}$$

First note that if export supplies are perfectly elastic (i.e., $\mu_{g,c} \to \infty$), then equation (19) becomes equation (6) as before. It is also straightforward that assuming upward sloping supply curves will lead to lower trade creation than when assuming perfectly elastic supply curves (i.e., the term in brackets is smaller than 1). The intuition is that with upward sloping supply curves the world price of good g from country c increase as demand for this good increases, therefore reducing demand quantities.

Trade diversion

It is our understanding that the current formula for trade diversion in SMART when export supplies are assumed to be upward sloping is given by (14) above. It is clear however that the formula needs to be modified to consider changes in world prices of both good c and good $\neq c$, associated with a trade preference granted to good g when

imported from country c. More formally, equation (10) will change because both world prices will be affected.

Using (18), equation (10) becomes:

$$d\left(\frac{p_{g,c}^d}{p_{g,\neq c}^d}\right)\Bigg/ \frac{p_{g,c}^d}{p_{g,\neq c}^d} = \frac{\dfrac{p_{g,c}^w dt_{g,c} + dp_{g,c}^w (1+t_{g,c})}{p_{g,\neq c}^w (1+t_{g,\neq c})} - p_{g,c}^w (1+t_{g,c})\dfrac{dp_{g,\neq c}^w (1+t_{g,\neq c})}{[p_{g,\neq c}^w (1+t_{g,\neq c})]^2}}{\dfrac{p_{g,c}^w (1+t_{g,c})}{p_{g,\neq c}^w (1+t_{g,\neq c})}} \qquad (21)$$

Further substituting (17) into (21), and recalling that initial world prices have been normalized to 1, we obtain:

$$d\left(\frac{p_{g,c}^d}{p_{g,\neq c}^d}\right)\Bigg/ \frac{p_{g,c}^d}{p_{g,\neq c}^d} = \frac{dt_{g,c}}{(1+t_{g,c})} + \frac{dx_{g,c}}{x_{g,c}\mu_{g,c}} - \frac{dx_{g,\neq c}}{x_{g,\neq c}\mu_{g,\neq c}} \qquad (22)$$

By definition of trade diversion $dx_{g,c} = -dx_{g,\neq c}$. Then equation (22) becomes:

$$d\left(\frac{p_{g,c}^d}{p_{g,\neq c}^d}\right)\Bigg/ \frac{p_{g,c}^d}{p_{g,\neq c}^d} = \frac{dt_{g,c}}{(1+t_{g,c})} + dx_{g,c}\left(\frac{1}{x_{g,c}\mu_{g,c}} + \frac{1}{x_{g,\neq c}\mu_{g,\neq c}}\right) \qquad (23)$$

By definition $dm_{g,c} \equiv -dm_{g,\neq c} \equiv dx_{g,c} \equiv dx_{g,\neq c}$. Then substituting (23) and (11) into (9), and solving for $dm_{g,c}$ yields:

$$TD_{g,c} = \frac{m_{g,c}m_{g,\neq c}}{m_{g,c} + m_{g,\neq c}} \frac{dt_{g,c}}{1+t_{g,c}} \sigma_{g,c,\neq c}\left[\frac{(m_{g,c} + m_{g,\neq c})\mu_{g,c}\mu_{g,\neq c}}{(m_{g,c} + m_{g,\neq c})\mu_{g,c}\mu_{g,\neq c} - m_{g,c}\mu_{g,c} - m_{g,\neq c}\mu_{g,\neq c}}\right] \qquad (24)$$

It is easy to verify that if the elasticities of export supply are both perfectly elastic, then equation (24) becomes equation (12). It is also clear that the extent of trade diversion is null if the export supply of c or $\neq c$ is perfectly inelastic ($\mu_{g,c} = 0$ and/or $\mu_{g,\neq c} = 0$). Also trade diversion defined in (24) is than trade diversion defined in (12).

One potentially relevant scenario is when the elasticity of export supply of the rest of the world ($\neq c$) is infinitely elastic, but not that of the trading partner to which the preference is granted (c). Then equation (24) becomes:

$$TD_{g,c} = \frac{m_{g,c} m_{g,\neq c}}{m_{g,c} + m_{g,\neq c}} \frac{dt_{g,c}}{1 + t_{g,c}} \sigma_{g,c,\neq c} \left[\frac{(m_{g,c} + m_{g,\neq c})\mu_{g,c}}{(m_{g,c} + m_{g,\neq c})\mu_{g,c} - m_{g,\neq c}} \right] \qquad (25)$$

Constraining trade diversion

Again, by definition, trade diversion cannot be larger than the original imports from the rest of the world ($\neq c$). Thus the expressions in (24) or (25) would have to be constrained to satisfy this. The approach we prefer is the one used in equation (13) earlier, where this constrained is imposed on the data after calculating trade-diversion for good g. So under the assumptions of equation (25), trade diversion would be defined as:

$$TD_{g,c} = \begin{cases} \dfrac{m_{g,c} m_{g,\neq c}}{m_{g,c} + m_{g,\neq c}} \dfrac{dt_{g,c}}{1 + t_{g,c}} \sigma_{g,c,\neq c} \left[\dfrac{(m_{g,c} + m_{g,\neq c})\mu_{g,c}}{(m_{g,c} + m_{g,\neq c})\mu_{g,c} - m_{g,\neq c}} \right] & \text{if } dm_{g,c} \leq m_{g,\neq c} \\ m_{g,\neq c} & \text{if } dm_{g,c} > m_{g,\neq c} \end{cases} \qquad (26)$$

Alternatively, one could follow the procedure embedded in SMART which ensures that this is satisfy when the preferential tariff reduction goes to infinity. This simply involves adding a term in the denominator of (24) or (25), so that when the change in preferential tariffs goes to infinity, trade diversion tends to $m_{g,c}$. In the case of the assumption leading to equation (25) this translates into transforming equation (25) as follows:

$$TD_{g,c} = \frac{m_{g,c} m_{g,\neq c} \dfrac{dt_{g,c}}{1 + t_{g,c}} \sigma_{g,c,\neq c} \left[\dfrac{(m_{g,c} + m_{g,\neq c})\mu_{g,c}}{(m_{g,c} + m_{g,\neq c})\mu_{g,c} - m_{g,\neq c}} \right]}{m_{g,c} + m_{g,\neq c} + m_{g,c} \dfrac{dt_{g,c}}{1 + t_{g,c}} \sigma_{g,c,\neq c} \left[\dfrac{(m_{g,c} + m_{g,\neq c})\mu_{g,c}}{(m_{g,c} + m_{g,\neq c})\mu_{g,c} - m_{g,\neq c}} \right]} \qquad (27)$$

Again as before the total increase in exports of good g from country c after the increase in tariff preferences is given by the sum of the trade diversion and trade creation terms. One

could further sum these effects across goods and countries (if several country cs) are involved.

Modeling world markets: GSIM

As mentioned earlier world markets are ignored in SMART (which is theoretically reasonable if one assumes infinitely elastic export supply elasticities). Everything is treated in terms of bilateral relationships (with third countries exporting to the importing country). This is particularly awkward when assuming upward sloping export-supply functions. It indeed translates into assuming that exports of good g from country c to different markets are not substitutable. There are several ways in which world markets can be included into this: GSIM developed by Francois and Hall (2003) is one alternative and is the one offered by WITS.

Demand side

Let us define the import demand function of country m for product p exported by country x as a function of the domestic price in m of product p exported by x in, the domestic price in m of product p exported by other countries and the total expenditure of country m on product p (the latter results from the assumption of weakly separable import demand functions):

$$M_{m,p,x} = f\left(P_{m,p,x}; P_{m,p,\neq x}; Y_{m,p}\right) \tag{1}$$

where $Y_{m,p}$ is the total import expenditure of country m on product p, and $P_{m,p,x}$ is the domestic price in country m (tariff inclusive) of product p exported by x, i.e., $P_{m,p,x} \equiv \left(1 + t_{m,p,x}\right)P^*_{p,x} \equiv T_{m,p,x}P^*_{p,x}$, where $t_{m,p,x}$ is the preference inclusive tariff imposed by country m on its imports of p from x, and $P^*_{p,x}$ is the world price of product p exported from x.

By differentiating equation (1), applying the Slutsky decomposition of partial demand, and taking advantage of the zero homogeneity property of Hicksian demand, we can then derive the following:

$$\varepsilon_{m,p,x} = \theta_{m,p,x}\left(\varepsilon_{m,p} + \sigma_{m,p}\right)$$

$$\varepsilon_{m,p,\neq x} = \theta_{m,p,\neq x}\varepsilon_{m,p} - \left(1 - \theta_{m,p,\neq x}\right)\sigma_{m,p} \tag{2}$$

where $\theta_{m,p,x}$ is the expenditure share of product p exported by x in total imports of product p by country m, $\varepsilon_{m,p} < 0$ is the composite import demand function for product p in country m, $\sigma_{m,p} > 0$ is the elasticity of substitution in country m for product p exported from different countries, $\varepsilon_{m,p,x}$ is the import demand function in country m for product p exported from x, and $\varepsilon_{m,p,\neq x}$ is the cross price elasticity of the import demand function in country m for product p exported from x, when the price of product p exported from other countries $\neq x$ changes.

Supply side

Similarly, we define export supply functions to be a function of world prices:

$$X_{p,x} = g\left(P_{p,x}^*\right) \tag{3}$$

Differentiating (3), and rearranging in percentage terms we obtain the definition of the export supply elasticity:

$$e_{p,x} = \hat{X}_{p,x}\big/\hat{P}_{p,x}^* > 0 \tag{4}$$

Market Equilibrium

Once demand and supply for each product are specified we can solve for the percentage world price increase following a trade reform in one or more countries by simply solving for the new price that re-equilibrates demand and supply for this product. Because of imperfect substitution when tariffs on products exported by other countries change, this will also affect import demand as suggested by the cross-price elasticity in equation (2).

Matrix notation will help us obtain a quick analytical solution to changes in world prices following trade policy reforms. First denote $\mathbf{E_{m,p}}$ as a diagonal x by x matrix of elasticities in country m for product p, where the elements in the diagonal are equal to

$\varepsilon_{m,p,x}/e_{p,x}$ as provided by (2) and (4) and the elements off the diagonal are given by:

$\varepsilon_{m,p,\neq x}/e_{p,x}$. Denote $\mathbf{P_p^*}$ as a vector of percentage changes in world prices of product p

and $\mathbf{T_{m,p}}$ a vector of changes on the tariff imposed by country m on imports of p from

different countries. Further denote $\mathbf{E_p} = \sum_m \mathbf{E_{m,p}}$ and $\mathbf{B_p} = \sum_m \mathbf{E_{m,p}T_{m,p}}$. Imposing the

market clearing conditions and solving for the changes in world prices yields:

$$\mathbf{P_p^*} = \left(\mathbf{I} - \mathbf{E_p}\right)^{-1}\mathbf{B_p} \tag{5}$$

Once we obtain the percentage changes in world prices using equation (5) it is trivial to

obtain the changes in import and export revenue, tariff revenue, (import) consumer

surplus and (export) producer surplus, and therefore welfare.

Computing changes in revenue and welfare

The change in export revenue can be obtained using equation (4) once the percentage

change in world price is obtained using equation (5). A linear approximation to the

change in (exporter) producer surplus is given by:

$$\Delta PS_{p,x} = P_{p,x}^* X_{p,x} \hat{P}_{p,x}^* \left(1 + \frac{e_{p,x}\hat{P}_{p,x}^*}{2}\right)$$

where $\hat{P}_{p,x}^*$ is the percentage change in the world price of good p *exported from* x.

The percentage change in imports can be obtained using (2) and the definition of import

demand elasticity (i.e., the ratio of percentage quantity changes over percentage price

changes).

The linear approximation to changes in tariff revenue is given by:

$$\Delta TR_{m,p,x} = t_{m,p,x} M_{m,p,x} P_{m,p,x}^* \left((\hat{t}_{m,p,x}) + \hat{P}_{m,p,x}^* \left(1 + \varepsilon_{m,p,x}\right)\right)$$

where $\hat{t}_{m,p,x}$ *is the percentage change in the tariff imposed by* m *on good* p *exported from*

x.

For consumer surplus, assume a CES aggregator across different export sources. The change in consumer surplus is then given by:

$$\Delta CS_{m,p} = \sum_{x} M_{m,p,x} P^{*}_{m,p,x} T_{m,p,x} \left(\frac{1}{2} \varepsilon_{m,p} \left[\hat{P}_{m,p} \right]^{2} \mathbf{sign}\left(\hat{P}_{m,p} \right) - \hat{P}_{m,p} \right)$$

where $\hat{P}_{m,p} = \sum_{x} \theta_{m,p,x} \hat{P}^{*}_{p,x} + \hat{T}_{p,x}$

Finally, the change in welfare is given by the sum by country of the changes in producer surplus, consumer surplus and tariff revenue.

References

Francois, Joseph and Keith Hall (2003), "Global simulation analysis of industry-level trade policy", mimeo, The World Bank.